# Projectile Motion Experiment

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# Contents

# 1 Task of the Experiment

The mission of the experiment is to find the following,

-Flight time of the projectile.

-Maximum range of the projectile.

-Maximum altitude of the projectile.

# 2 Theory

In order to find the answers to the given tasks we shall derive formulas from the experiment environment.
An environment of a projectile motion can be interpreted by the initial velocity of the projectile($v_0$) and the two distance variables of $x - y$ axis, height($h$) and range($x$).

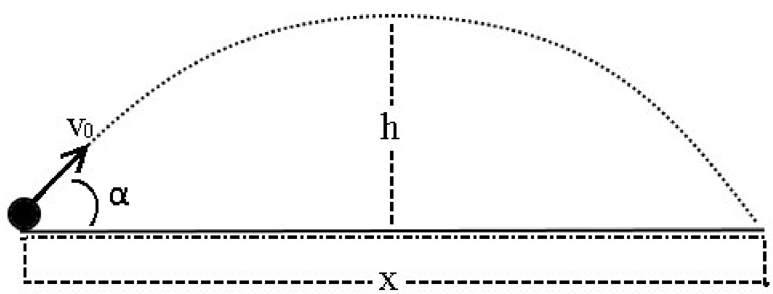


Figure 1: Diagram

In order to analyse the situation more exhaustively, we shall allocate the velocity to its vectors throughout its motion in the system.
With this we get the vectors $v_0 sin\alpha$ and $v_0 cos\alpha$ as a counterpart for $x - y$ axis.

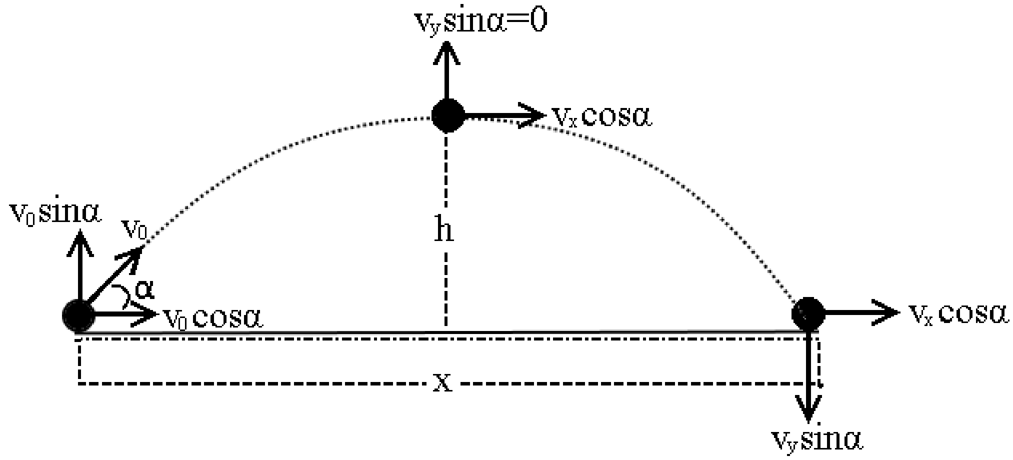


Figure 2: Diagram

In accordance with our previous information,

$$\boldsymbol{r = r_0 + v_0 t + \frac{1}{2}at^2}$$

We can arrange this distance formula according to our needs. Our initial distance is 0, therefore $\boldsymbol{r_0 = 0}$. Considering that h is on y plane, we write the y vector of $\boldsymbol{v_0}$ as $\boldsymbol{v_0 sin\alpha}$. And use $\boldsymbol{h}$ to indicate height instead of r.

$$\boldsymbol{h = h_0 + v_0 sin\alpha t - \frac{1}{2}gt^2}$$

By implementing the same idea to x vector, $\boldsymbol{r_0 = 0}$ wit addition of no acceleration on x vector .

$$\boldsymbol{x = v_0 cos\alpha t}$$

## 2.1 Flight Time

### 2.1.1 Flat Field

Time of flight on a flat experiment surface is as follows.

$$v_y = v_0 sin\alpha - gt$$

Object at its peak on y axis has the $v_y$ velocity of 0. Therefore,

$$0 = v_0 sin\alpha - gt$$

After we arrange the equation in order to find t,

$$t = \frac{v_0 sin\alpha}{g}$$

But it is important to not to forget this gives the time until its peak point on y axis. Due to parabolas symmetric behaviour we can say the total time is the double of this, which give us,

$$\boxed{t = \frac{2v_0 sin\alpha}{g}}$$

### 2.1.2 Elevated Field

By taking y as a function of t,
$y(t) = y_0 + v_0 sin\alpha t - \frac{1}{2}gt^2$

Set the position of the projectile to 0 to find time, $y(t) = 0$
$0 = y_0 + v_0 sin\alpha t - \frac{1}{2}gt^2$

$$-\frac{1}{2}gt^2 + v_0 sin\alpha t + y_0 = 0$$

By using the *Quadratic Formula*,

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-v_0 sin\alpha \pm \sqrt{(v_0 sin\alpha)^2 - 4(-\frac{1}{2})gy_0}}{-g}$$

Considering that the square root is positive with also velocity and sin value, we can continue with the positive value of t.

$$\boxed{t = \frac{v_0 sin\alpha + \sqrt{(v_0 sin\alpha)^2 + 2gy_0}}{g}}$$

## 2.2 Maximum Height

We know that,

$$\boldsymbol{h = h_0 + v_0 sin\alpha t - \frac{1}{2}gt^2}$$

To find the maximum height of the projectile we shall examine its position and vectors on its peak(Fig.2).
We have already found the time it takes to reach its peak in 2.1, which is:

$$\boldsymbol{t = \frac{v_0 sin\alpha}{g}}$$

By implementing the time when $\boldsymbol{v_y = 0}$ to our equation we can find find the maximum height.

$$\boldsymbol{h = h_0 + v_0 sin\alpha \frac{v_0 sin\alpha}{g} - \frac{1}{2}g(\frac{v_0 sin\alpha}{g})^2}$$

$$\boldsymbol{h = h_0 + \frac{(v_0 sin\alpha)^2}{g} - \frac{(v_0 sin\alpha)^2}{2}g}$$

$$\boxed{\boldsymbol{h = h_0 + \frac{(v_0 sin\alpha)^2}{2g}}}$$

## 2.3 Maximum Range

### 2.3.1 Flat Field

The distance over x axis in time is,

$$x = v_0 cos\alpha t$$

The total time of the motion is equal to the flight time.

$$t = \frac{2v_0 sin\alpha}{g}$$

If we implement the total time to our distance formula we get the maximum range.

$$x = v_0 cos\alpha \frac{2v_0 sin\alpha}{g}$$

$$x = \frac{2sin\alpha cos\alpha (v_0)^2}{g}$$

From the *Half Angle Formula*,

$$2sin\alpha cos\alpha = sin(2\alpha)$$

Therefore,

$$\boxed{x = \frac{sin(2\alpha)(v_0)^2}{g}}$$

### 2.3.2 Elevated Field

If the projectile is thrown from an altitude than we shall use the time formula from **Section 2.1.2**.

$$x = v_0 cos\alpha t$$

$$\boxed{x = \frac{v_0 cos\alpha}{g}[v_0 sin\alpha + \sqrt{(v_0 sin\alpha)^2 + 2gy_0}]}$$

# 3 Experiment

Experiment Experiment will be conducted with three different variables of influence which are Experiment will be conducted with three different variables of influence which are *velocity*, *angle*($\boldsymbol{\alpha}$) and *initial height of projectile*($\boldsymbol{h_0}$). With this values we will find,

-Flight time
-Maximum Height
-Maximum Range

of the projectile.

In the the experiments, values will be assigned to these variables. First the theoretic solutions will be written, then the computer simulations will be given after.

Gravitational acceleration $g$ will be taken as $\boldsymbol{g = 9.8}$ during all three experiments.

## 3.1 Experiment I

$\boldsymbol{v_0 = 10}$
$\boldsymbol{\alpha = 53^\circ}$
$\boldsymbol{h_0 = 0}$

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$\boldsymbol{sin\alpha = 0,798^\circ cos\alpha = 0,601^\circ}$

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**Flight time:**
Considering that $\boldsymbol{h_0 = 0}$,

$\boldsymbol{t = \frac{2v_0 sin\alpha}{g}}$

$\boldsymbol{t = \frac{2\cdot 10 sin53}{g} = \frac{2\cdot 10\cdot 0,798^\circ}{9,8}}$

$\boldsymbol{t = 1,628}$
**Maximum Height:**

$\boldsymbol{h = h_0 + \frac{(v_0 sin\alpha)^2}{2g}}$

$\boldsymbol{h = 0 + \frac{(10\cdot 0,798^\circ)^2}{2\cdot 9,8}}$

$\boldsymbol{h = 3,249}$
**Maximum Range:**

$\boldsymbol{x = \frac{sin(2\alpha)(v_0)^2}{g} = \frac{sin(2\cdot 53^\circ)(10)^2}{9,8}}$

$\boldsymbol{x = 9,808}$

## 3.2 Experiment II

$v_0 = 27$
$\alpha = 25°$
$h_0 = 9$

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$sin\alpha = 0,422° cos\alpha = 0,906°$

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**Flight time:**

$t = \frac{v_0 sin\alpha + \sqrt{(v_0 sin\alpha)^2 + 2gy_0}}{g}$

$t = \frac{27 \cdot sin25° + \sqrt{(27 sin25°)^2 + 2 \cdot g \cdot 9}}{g}$

$t = \frac{27 \cdot 0,422 + \sqrt{(27 \cdot 0,422)^2 + 2 \cdot 9,8 \cdot 9}}{9,8}$

$t = 2,948$

**Maximum Height:**

$h = h_0 + \frac{(v_0 sin\alpha)^2}{2g}$

$h = 9 + \frac{(27 \cdot sin25°)^2}{2g}$

$h = 9 + \frac{(27 \cdot 0,422)^2}{2 \cdot 9,8}$

$h = 15,623$

**Maximum Range:**

$x = \frac{v_0 cos\alpha}{g}[v_0 sin\alpha + \sqrt{(v_0 sin\alpha)^2 + 2gy_0}]$

$x = \frac{27 \cdot cos25°}{g}[27 \cdot sin25° + \sqrt{(27 \cdot sin21°)^2 + 2 \cdot g \cdot 9}]$

$x = \frac{27 \cdot 0,906}{9,8}[27 \cdot 0,422 + \sqrt{(27 \cdot 0,422)^2 + 2 \cdot 9,8 \cdot 9}]$

$x = 72,117$

### 3.3 Experiment III

$v_0 = 15$
$\alpha = 72^\circ$
$h_0 = 3$

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$sin\alpha = 0,951^\circ cos\alpha = 0,309^\circ$

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**Flight time:**

$t = \frac{v_0 sin\alpha + \sqrt{(v_0 sin\alpha)^2 + 2gy_0}}{g}$

$t = \frac{15 \cdot sin72^\circ + \sqrt{(15 sin72^\circ)^2 + 2 \cdot g \cdot 3}}{g}$

$t = \frac{15 \cdot 0,951 + \sqrt{(15 \cdot 0,951)^2 + 2 \cdot 9,8 \cdot 3}}{9,8}$

$t = 3,108$

**Maximum Height:**

$h = h_0 + \frac{(v_0 sin\alpha)^2}{2g}$

$h = 3 + \frac{(15 \cdot sin72^\circ)^2}{2g}$

$h = 3 + \frac{(15 \cdot 0,951)^2}{2 \cdot 9,8}$

$h = 13,382$

**Maximum Range:**

$x = \frac{v_0 cos\alpha}{g}[v_0 sin\alpha + \sqrt{(v_0 sin\alpha)^2 + 2gy_0}]$

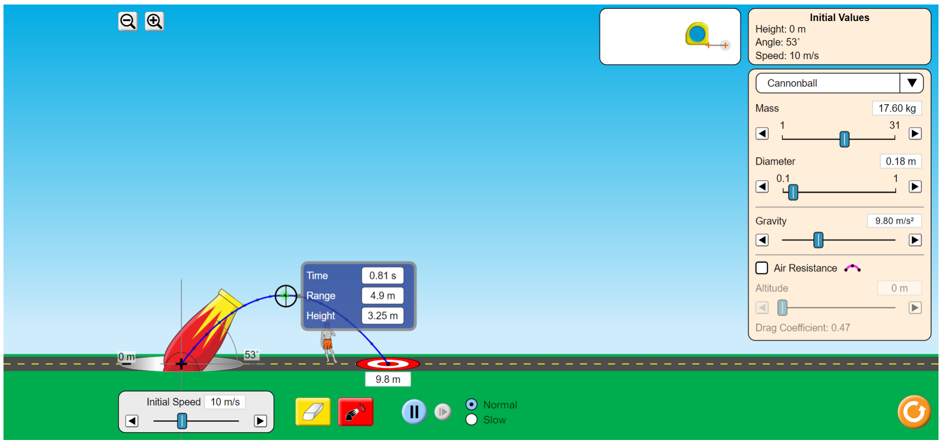
$x = \frac{15 \cdot cos72^\circ}{g}[15 \cdot sin72^\circ + \sqrt{(15 \cdot sin72^\circ)^2 + 2 \cdot g \cdot 3}]$

$x = \frac{15 \cdot 0,309}{9,8}[15 \cdot 0,951 + \sqrt{(15 \cdot 0,951)^2 + 2 \cdot 9,8 \cdot 3}]$

$x = 14,377$

# 4 Simulation Results

## 4.1 Experiment I

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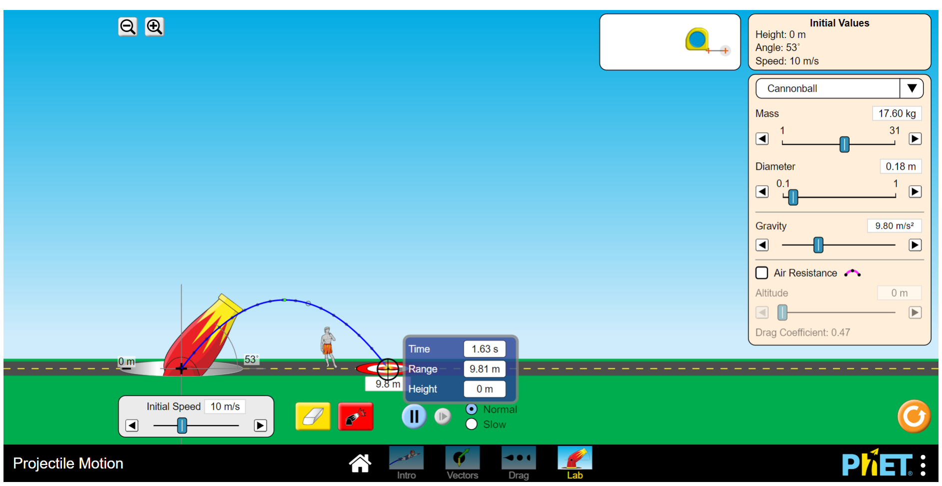
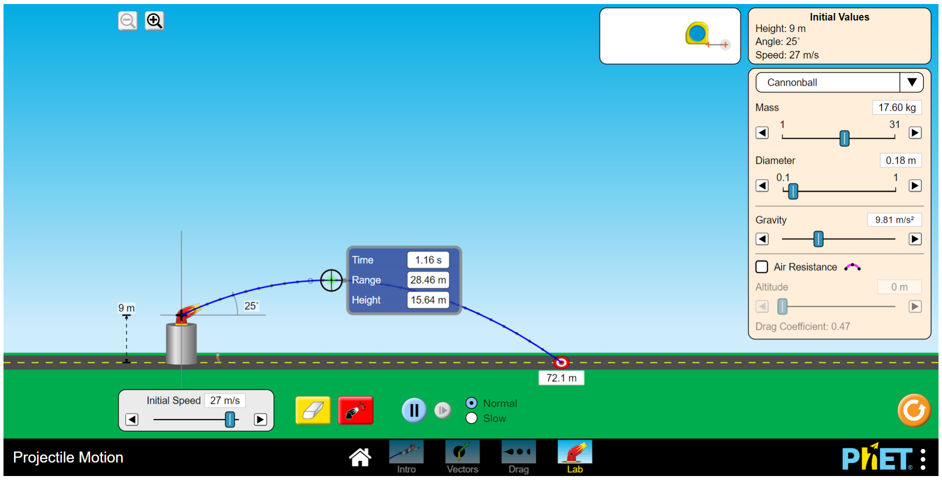



-Range and Time-

### 4.2 Experiment II

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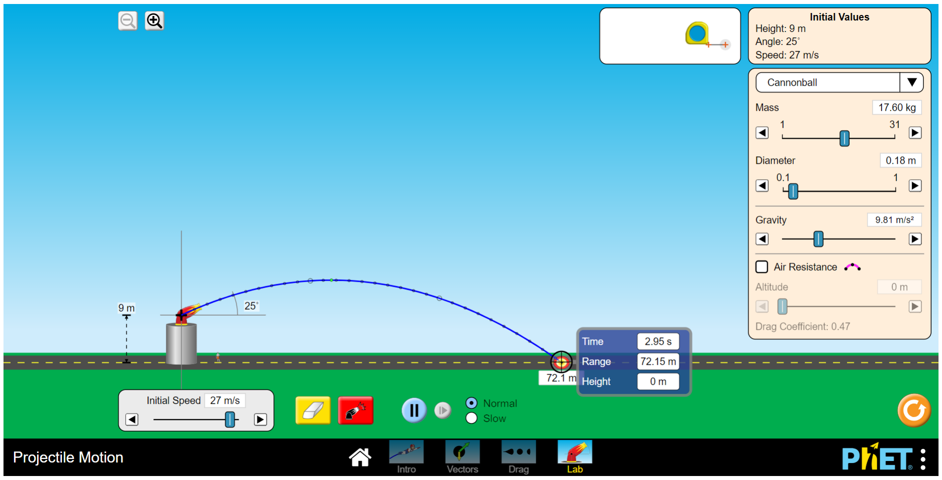
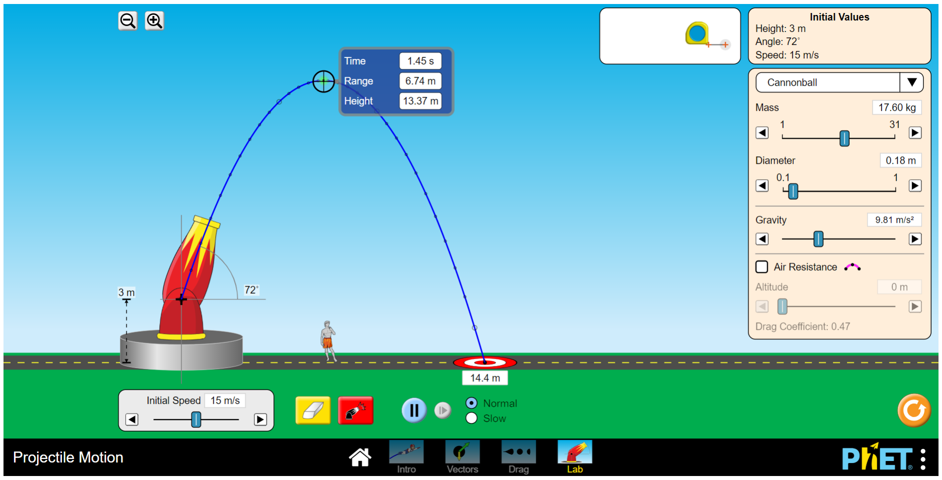



-Range and Time-

### 4.3 Experiment III

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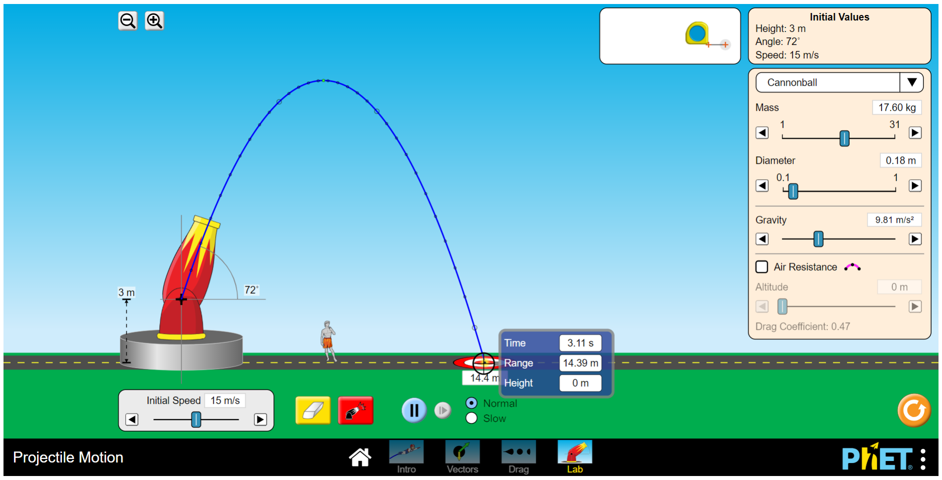


-Range and Time-

## 5 Conclusion

After comparing the result of my theoretical calculations and simulation the only difference between is that, computer rounds the result to nearest integer.
Rather than that I believe this simulation uses the same formulas to get the results therefore an experiment shall be done on earth to really compare the theory to real experiences.
In that case some other side-activities shall be taken to consideration such as wind, air resistance, gravity. If we want a precise measurement gravity needs to be measured due to fact that earth is not a perfect sphere therefore gravity has slight changes around globe and in this case altitude also has affect on both gravity and air resistance-due to changes in density of the air in atmosphere-.

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